What is a circle?
Where do you see circles?
What do you know about a circle?
What might be useful to know about a circle?

What You’ll Learn
• Measure the radius, diameter, and circumference of a circle.
• Investigate and explain the relationships among the radius, diameter, and circumference of a circle.
• Develop formulas to find the circumference and area of a circle.
• Estimate the measures for a circle, then use the formulas to calculate these measures.
• Draw a circle, given its radius, area, or circumference.
• Develop formulas to find the surface area and volume of a cylinder.

Why It’s Important
Measuring circles is an extension of measuring polygons in earlier grades.
Key Words

- diameter
- radius
- circumference
- irrational number
Rounding Measurements

1 cm = 10 mm
and, 1.6 cm = 16 mm

When a measurement in centimetres has 1 decimal place, the measurement is given to the nearest millimetre.

1 m = 100 cm
1.3 m = 130 cm
1.37 m = 137 cm

When a measurement in metres has 2 decimal places, the measurement is given to the nearest centimetre.

1 m = 1000 mm
1.4 m = 1400 mm
1.43 m = 1430 mm
1.437 m = 1437 mm

When a measurement in metres has 3 decimal places, the measurement is given to the nearest millimetre.

Example

Write each measurement to the nearest millimetre.

a) 25.2 mm

b) 3.58 cm

Solution

a) 25.2 mm
Since there are 2 tenths, round down.
25.2 mm is 25 mm to the nearest millimetre.

b) 3.58 cm
To round to the nearest millimetre, round to 1 decimal place.
Since there are 8 hundredths, round up.
3.58 cm is 3.6 cm to the nearest millimetre.

Check

1. a) Round to the nearest metre.
   i) 4.38 m     ii) 57.298 m     iii) 158.5 cm

   b) Round to the nearest millimetre and nearest centimetre.
      i) 47.2 mm   ii) 47.235 cm   iii) 1.0579 m
6.1 Investigating Circles

Which attribute do these objects share?

Work with a partner.
You will need circular objects, a compass, ruler, and scissors.
➢ Use a compass. Draw a large circle.
   Use a ruler.
   Draw a line segment that joins two points on the circle.
   Measure the line segment. Label the line segment with its length.
   Draw and measure other segments that join two points on the circle.
   Find the longest segment in the circle.
   Repeat the activity for other circles.

➢ Trace a circular object.
   Find a way to locate the centre of the circle.
   Measure the distance from the centre to the circle.
   Measure the distance across the circle, through its centre.
   Record the measurements in a table.
   Repeat the activity with other circular objects.
   What pattern do you see in your results?

Reflect & Share
Compare your results with those of another pair of classmates.
Where is the longest segment in any circle?
What relationship did you find between the distance across a circle through its centre, and the distance from the centre to the circle?
A circle is a closed curve.
All points on a circle are the same distance from the centre of the circle.
This distance is the **radius** of the circle.

The distance across a circle through its centre is the **diameter** of the circle.
The radius is one-half the diameter.
Let \( r \) represent the radius, and \( d \) the diameter.
Then, \( r = \frac{1}{2}d \), or \( r = \frac{d}{2} \)
The diameter is two times the length of the radius.
That is, \( d = 2r \)

**Example**

Draw a line segment.
Construct a circle for which this segment is:
a) the radius

**Solution**

a) Draw a line segment.
   Place the compass point at one end.
   Place the pencil point at the other end.
   Draw a circle.

b) Measure the line segment in part a.
   Draw a congruent segment.
   Mark its midpoint.
   Place the compass point at the midpoint.
   Place the pencil point at one end of the segment.
   Draw a circle.

**Practice**

1. Draw a circle with radius 6 cm.
   What is the diameter of the circle? Explain.

2. Draw a circle with diameter 8 cm.
   What is the radius of the circle? Explain.
How are the diameter and radius of a circle related? When you know the diameter, how can you find the radius? When you know the radius, how can you find the diameter? Include examples in your explanation.
You know the relationship between the radius and diameter of a circle. You will now investigate how these two measures are related to the distance around a circle.

Work with a partner. You will need 6 circular objects, dental floss, scissors, and a ruler.

Choose an object. Use dental floss to measure the distance around it. Measure the radius and diameter of the object. Record these measures in a table.

Repeat the activity for the remaining objects. What patterns can you see in the table? How is the diameter related to the distance around? How is the radius related to the distance around?

For each object, calculate:
- distance around / diameter
- distance around / radius

What do you notice?

Reflect & Share
Compare your results with those of another pair of classmates. Work together to write a formula for the distance around a circle, when you know its diameter.

The distance around a circle is its circumference. For any circle, the circumference, C, divided by the diameter, d, is \( \frac{C}{d} \), which is constant with value approximately 3.
So, the circumference is approximately 3 times the diameter. And, the circumference \( C \) divided by the radius, \( r \), is \( \frac{C}{r} \), which is approximately 6.

So, the circumference is approximately 6 times the radius.

For any circle, the ratio \( \frac{C}{d} = \pi \)

We use the symbol \( \pi \) because the value of \( \frac{C}{d} \) is an irrational number; that is, \( \pi \) represents a decimal that never repeats and never terminates.

The symbol \( \pi \) is a Greek letter that we read as “pi.”

\[ \pi \approx 3.14 \]

So, the circumference is \( \pi \) multiplied by \( d \).

We write: \( C = \pi d \)

Since \( d = 2r \), the circumference is also \( \pi \) multiplied by \( 2r \).

We write: \( C = \pi \times 2r \), or \( C = 2\pi r \)

When we know the radius or diameter of a circle, we can use one of the formulas above to find the circumference of the circle.

**Example 1**

The face of a toonie has a radius of 1.4 cm.

a) What is the diameter of the face?

b) Estimate the circumference of the face.

c) Calculate the circumference.

Give the answer to the nearest millimetre.

**Solution**

a) The diameter \( d = 2r \), where \( r \) is the radius.

Substitute: \( r = 1.4 \)

\[ d = 2 \times 1.4 \]

\[ = 2.8 \]

The diameter is 2.8 cm.

b) The circumference is approximately 3 times the diameter:

\[ 3 \times 2.8 \text{ cm} \approx 3 \times 3 \text{ cm} \]

\[ = 9 \text{ cm} \]

The circumference is approximately 9 cm.
c) Method 1
The circumference is: \( C = \pi d \)
Substitute: \( d = 2.8 \)
\[ C = \pi \times 2.8 \]
Key in: \( \pi \times 2.8 \) to display 8.79645943
So, \( C \approx 8.796 \)
\[ \approx 8.8 \]
The circumference is 8.8 cm, to the nearest millimetre.

Method 2
The circumference is: \( C = 2\pi r \)
Substitute: \( r = 1.4 \)
\[ C = 2 \times \pi \times 1.4 \]
Key in: \( 2 \times \pi \times 1.4 \) to display 8.79645943
So, \( C \approx 8.796 \)
\[ \approx 8.8 \]
The circumference is 8.8 cm, to the nearest millimetre.

When we know the circumference, we can use a formula to find the diameter. Use the formula \( C = \pi d \). To isolate \( d \), divide each side by \( \pi \).
\[
\frac{C}{\pi} = \frac{\pi d}{\pi}
\]
\[ \frac{C}{\pi} = d \]
So, \( d = \frac{C}{\pi} \)

Example 2
A circular pond has circumference 12 m.

a) Estimate the lengths of the diameter and radius of the pond.

b) Calculate the diameter and radius.
Give the answers to the nearest centimetre.

Solution
a) Since the circumference is approximately 3 times the diameter, then the diameter is about \( \frac{1}{3} \) the circumference.
One-third of 12 m is 4 m. So, the diameter is about 4 m.
The radius is \( \frac{1}{2} \) the diameter. One-half of 4 m is 2 m.
So, the radius of the pond is about 2 m.

b) The diameter is: \( d = \frac{C}{\pi} \)
Substitute: \( C = 12 \)
\[ d = \frac{12}{\pi} \]
\[ \approx 3.8197 \]
Use a calculator.
Do not clear the calculator.
The radius is \( \frac{1}{2} \) the diameter.
Divide the number in the calculator display by 2.
\[ r \approx 1.9099 \]
The diameter is 3.82 m to the nearest centimetre.
The radius is 1.91 m to the nearest centimetre.
6.2 Circumference of a Circle

1. Estimate the circumference of each circle.
   a) b) c)
   
2. Calculate the circumference of each circle in question 1.
   Give the answers to 1 decimal place.

3. Estimate the diameter and radius of each circle.
   a) b) c)
   
4. Calculate the diameter and radius of each circle in question 3.
   Give the answers to the nearest millimetre.

5. When you estimate the circumference, you use 3 instead of $\pi$. Is the estimated value greater than or less than the calculated value? Explain.

6. A circular garden has diameter 2.4 m.
   a) The garden is to be enclosed with plastic edging. How much edging is needed?
   b) The edging costs $4.53/m. What is the cost to edge the garden?

Science
The orbit of Earth around the sun is approximately a circle.
The radius of the orbit is about $1.5 \times 10^8$ km.
How could you calculate the circumference of the orbit?
The orbit of a communication satellite around Earth is approximately a circle.
The satellite is about 35 800 km above Earth’s surface.
What would you need to know to be able to calculate the radius of the orbit?
7. **Assessment Focus** A bicycle tire has a stone stuck in it. The radius of the tire is 46 cm. Every time the wheel turns, the stone hits the ground.
   a) How far will the bicycle travel between the stone hitting the ground for the first time and the second time?
   b) How many times will the stone hit the ground when the bicycle travels 1 km?
Show your work.

8. A carpenter is making a circular tabletop with circumference 4.5 m. What is the radius of the tabletop in centimetres?

9. Can you draw a circle with circumference 33 cm? If you can, draw the circle and explain how you know its circumference is correct. If you cannot, explain why it is not possible.

10. a) What if you double the diameter of a circle. What happens to the circumference?
    b) What if you triple the diameter of a circle. What happens to the circumference?

11. Suppose a metal ring could be placed around Earth at the equator.
   a) The radius of Earth is 6378.1 km. How long is the metal ring?
   b) Suppose the length of the metal ring is increased by 1 km. Would you be able to crawl under the ring, walk under the ring, or drive in a school bus under the ring? Explain how you know.

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**Reflect**

When you know the circumference of a circle, how can you calculate its radius and diameter? Include an example in your explanation.
6.3 Area of a Circle

**Focus**
Develop and use the formula for the area of a circle.

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**Explore**

Work in a group of 4.

You will need scissors, glue, a compass, and protractor.

➢ Each of you draws a circle with radius 8 cm.
  Each of you chooses one of these:
  - 4 congruent sectors
  - 8 congruent sectors
  - 10 congruent sectors
  - 12 congruent sectors

➢ Use a protractor to divide your circle into the number of sectors you chose.
  Cut out the sectors.
  Arrange the sectors to approximate a parallelogram.

Glue the sectors on paper.

Calculate the area of the parallelogram.

Estimate the area of the circle.

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**Reflect & Share**

Compare your answer for the area of the circle with those of your group members.

Which area do you think is closest to the area of the circle? Explain.

How could you improve your estimate for the area?
A circle, with radius 10 cm, was cut into 24 congruent sectors. The sectors were then arranged to form a parallelogram.

The more congruent sectors we use, the closer the area of the parallelogram is to the area of the circle. If the number of sectors is large enough, the parallelogram is almost a rectangle.

The sum of the two longer sides of the rectangle is equal to the circumference, $C$.

$$C = 2\pi r$$

$$= 2\pi \times 10 \text{ cm}$$

$$= 20\pi \text{ cm}$$

So, each longer side is: $\frac{20\pi \text{ cm}}{2} = 10\pi \text{ cm}$

Each of the two shorter sides is equal to the radius, 10 cm.

The area of the rectangle is: $10\pi \text{ cm} \times 10 \text{ cm} = 10^2\pi \text{ cm}^2$

$$= 100\pi \text{ cm}^2$$

So, the area of the circle with radius 10 cm is $100\pi \text{ cm}^2$.

We can apply this idea to a circle with radius $r$. 

A circle, with radius 10 cm, was cut into 24 congruent sectors. The sectors were then arranged to form a parallelogram.
So, the area, \( A \), of a circle with radius \( r \) is: \( \pi r \times r = \pi r^2 \)

We can use this formula to find the area of any circle when we know its radius.

**Example**

The face of a dime has diameter 1.8 cm.

**a)** Estimate the area of the face of the dime.

**b)** Calculate the area. Give the answer to 2 decimal places.

**Solution**

The diameter of the face of a dime is 1.8 cm.

So, its radius is:

\[
\frac{1.8 \text{ cm}}{2} = 0.9 \text{ cm}
\]

**a)** The area of the face of the dime is about \( 3 \times r^2 \).

\[
r \approx 1
\]

So, \( r^2 = 1 \)

and \( 3 \times r^2 = 3 \times 1 = 3 \)

The area of the face of the dime is approximately 3 cm\(^2\).

**b)** Use the formula: \( A = \pi r^2 \)

Substitute: \( r = 0.9 \)

\[
A = \pi \times 0.9^2
\]

Use a calculator.

Key in: \( \left( \pi \right) \times \frac{0.9 \times 0.9 \text{ ENTER}}{x^2} \) to display 2.544690049

\( A \approx 2.54 \text{ cm}^2 \)

The area of the face of the dime is 2.54 cm\(^2\) to 2 decimal places.

Since 1 mm = 0.1 cm

Then 1 mm\(^2\) = 1 mm \times 1 mm

\[
= 0.1 \text{ cm} \times 0.1 \text{ cm}
\]

\[
= 0.01 \text{ cm}^2
\]

This illustrates that when an area in square centimetres has 2 decimal places, the area is given to the nearest square millimetre. In the Example, the area 2.54 cm\(^2\) is written to the nearest square millimetre.
1. Estimate the area of each circle.
   a)  
   b)  
   c)  

2. Calculate the area of each circle in question 1.
   Give the answers to the nearest square millimetre.

3. a) Use the results of questions 1 and 2.
   What if you double the radius of a circle.
   What happens to its area?
   b) What do you think happens to the area of a circle
      if you triple its radius?
   Justify your answers.

4. Assessment Focus  
   Use 0.5-cm grid paper.
   Draw a circle with radius 5 cm.
   Draw a square outside the circle that just encloses the circle.
   Draw a square inside the circle so that its vertices lie on the circle.
   a) How can you use the areas of the two squares to estimate
      the area of the circle?
   b) Check your estimate in part a by calculating the area
      of the circle.
   c) Repeat the activity for circles with different radii.
      Record your results.
   Show your work.

5. In the biathlon, athletes shoot at targets.
   Each target is 50 m from the athlete.
   Find the area of each target.
   a) The target for the athlete who is standing is a
      circle with diameter 11.5 cm.
   b) The target for the athlete who is prone is a circle
      with diameter 4.5 cm.
   Give the answers to the nearest square centimetre.
6. a) A square has side length 1 cm.
   i) What is the area of the square in square centimetres?
   ii) What is the area of the square in square metres?
   iii) Use the results of parts i and ii to write 1 cm² in square metres.

b) A calculator display shows the area of a circle as 7.068583471 m².
   What is this area rounded to the nearest square centimetre?

7. In curling, the target area is a bull’s eye series of 4 concentric circles.

   a) Calculate the area of the smallest circle.
      Write the area to the nearest square centimetre.
   b) When a smaller circle overlaps a larger circle, a ring is formed.
      Calculate the area of each ring on the target area to the
      nearest square centimetre.

8. The bottom of a swimming pool is a circle with circumference 31.4 m.
   What is the area of the bottom of the pool?
   Give the answer to the nearest square metre.

Take It Further

9. A large pizza has diameter 35 cm.
   Two large pizzas cost $19.99.
   A medium pizza has diameter 30 cm.
   Three medium pizzas cost $24.99.
   Which is the better deal: 2 large pizzas or 3 medium pizzas?
   Justify your answer.

Reflect

When you know the radius of a circle, how can you calculate its area?
Include an example in your explanation.
LESSON 6.1
1. A circle has radius 3.6 cm. What is its diameter?

2. A circle has diameter 3.6 cm. What is its radius?

3. a) Draw a large circle. Label its centre C. Mark points P, Q, and R on the circle. Join P, Q, and R to form \( \triangle PQR \). Join QC and RC. These line segments form 2 angles at C. Measure \( \angle QPR \) and the smaller \( \angle QCR \). How are these angles related?
   
   b) Repeat part a for a different circle. Is the relationship in part a still true? Explain.

4. The face of a penny has radius 9.5 mm.
   a) Estimate the circumference of the penny.
   b) Calculate the circumference. Give the answer to the nearest tenth of a millimetre.

5. An auger is used to drill a hole in the ice, for ice fishing. The diameter of the hole is 25 cm. What is the circumference of the hole?

6. Explain how you could calculate the circumference of a paper plate.

7. There is a clock on the Peace Tower in Ottawa. The circumference of the clock face is approximately 15.02 m.
   a) Estimate the diameter and radius of the clock face.
   b) Calculate the diameter and radius of the clock face to the nearest centimetre.

8. a) How is the circumference of a circle with radius 9 cm related to the circumference of a circle with diameter 9 cm?
   b) Draw both circles in part a.

9. The radius of a circular tray is 14.4 cm. What is its area to the nearest square millimetre?

10. The diameter of a circle is 58 m. What is its area to the nearest square centimetre?

11. A circular table has radius 56 cm. A tablecloth covers the table. The edge of the cloth is 10 cm below the tabletop. What is the area of the tablecloth?

12. a) How is the area of a circle with radius 6 cm related to the area of a circle with diameter 6 cm?
    b) Draw both circles in part a. Do the diagrams justify your answer in part a? Explain.
A cylinder is formed when a circle is translated through the air so that the circle is always parallel to its original position.

How does this relate to the triangular prism in *Unit 3*, page 112?

**Explore**

Work with a partner.
You will need cylindrical objects and a ruler.
Choose a cylindrical object.
Calculate its volume.
How did you use the diameter and the radius in your calculations?
How did you use $\pi$?

**Reflect & Share**

Share your method for calculating the volume with another pair of classmates.
Work together to write a formula for the volume of a cylinder.
Use any of diameter, radius, height, and $\pi$ in your formula.

**Connect**

A cylinder is a prism.
The volume of a prism is base area $\times$ height.
A can of baked beans is cylindrical.
Its diameter is 7.4 cm. Its height is 10.5 cm.
To find the volume of the can, first find the area of its base.
The base is a circle, with diameter 7.4 cm.
So, its radius is: $\frac{7.4\text{ cm}}{2} = 3.7$ cm
The base area: $A = \pi r^2$
Substitute: $r = 3.7$
$A = \pi(3.7)^2$
The height of the can is 10.5 cm.
So, its volume is: 

\[ V = \text{base area} \times \text{height} \]

\[ = \pi (3.7)^2 \times 10.5 \]

Key in: \[ \pi \times 3.7 \times 10.5 \text{ Enter} \] to display 451.588236

\[ V \approx 451.6 \]

The volume of the can of baked beans is about 452 cm\(^3\).

We can apply this idea to write a formula for the volume of any cylinder.

Its radius is \( r \).

So, its base area is \( \pi r^2 \).

Its height is \( h \).

So, its volume is: 

\[ V = \text{base area} \times \text{height} \]

\[ = \pi r^2 \times h \]

\[ = \pi r^2 h \]

So, a formula for the volume of a cylinder is \( V = \pi r^2 h \),

where \( r \) is the radius of its base, and \( h \) its height.

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**Example**

The base of a juice can is a circle with diameter 6.8 cm.

The height of the can is 12.2 cm.

What is the volume of the can?

**Solution**

The radius of the base is: \( \frac{6.8 \text{ cm}}{2} = 3.4 \text{ cm} \)

Use the formula for the volume of a cylinder:

\[ V = \pi r^2 h \]

Substitute: \( r = 3.4 \) and \( h = 12.2 \)

\[ V = \pi (3.4)^2 \times 12.2 \]

Key in: \[ \pi \times 3.4 \times 12.2 \text{ Enter} \] to display 443.0650951

\[ V \approx 443.07 \]

The volume of the can is about 443 cm\(^3\).

Capacity is measured in litres or millilitres.

Since 1 cm\(^3\) = 1 mL, the capacity of the can in the *Example* is about 443 mL.
Give each volume to the nearest cubic unit.

1. Calculate the volume of each cylinder.
   a) Height 10 cm, radius 4 cm
   b) Height 50 mm, radius 15 mm
   c) Height 12.4 m, radius 2.9 m

2. A candle mould is cylindrical. Its radius is 5 cm and its height is 20 cm. What is the capacity of the mould?

3. Frozen apple juice comes in cylindrical cans. A can is 12 cm high and has radius 3.5 cm.
   a) What is the capacity of the can?
   b) Apple juice expands when it freezes. The can is filled to 95% of its volume. What is the volume of apple juice in the can?

4. A core sample of earth is cylindrical. The length of the core is 300 mm. Its diameter is 150 mm. Calculate the volume of earth in cubic millimetres and cubic centimetres.

5. **Assessment Focus** A concrete column in a parkade is cylindrical. The column is 10 m high and has diameter 3.5 m.
   a) What is the volume of concrete in one column?
   b) There are 127 columns in the parkade. What is the total volume of concrete?
   c) What if the concrete in part a is made into a cube. What would the dimensions of the cube be?

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**Calculator Skills**

Find the sum of the composite numbers between 1 and 40.

**Reflect**

How is the volume of a cylinder related to the volume of a triangular prism?
How are these volumes different?
Explaining Solutions

To explain a solution using only words helps to communicate why you did what you did. When you explain, show the numbers or models you used.

Here are two solutions to this problem:
Six students wrote a math test. Their mean mark was 68%.
Another student wrote the test and scored 89%.
What is the mean mark for the 7 students?

Jol explains:
I took the mean mark and multiplied it by 6, the number of students. This gave me the total marks for all of them. Next, I added the mark of the seventh student to get the total marks for everyone. I then divided the total marks by 7, the total number of students who took the test. This gave me the mean mark for the 7 students.

Jol writes:
\[
\text{(original mean)} \times \frac{(\text{number of students})}{\text{total number of students}} + \frac{(7\text{th mark})}{7} = \text{new mean}
\]
\[
\frac{68 \times 6 + 89}{7} = \frac{408 + 89}{7} = \frac{497}{7} = 71
\]
The mean mark for the 7 students is 71%.

Veronica explains:
I subtracted the mean from the mark of the 7th student. This told me how many marks above the mean the 7th student had. I shared these marks equally among the 7 students. Then I added the extra marks to the original mean to get the new mean.

Veronica writes:

89 \div 68 = 21
21 \text{ extra marks} \div 7 = 3 \text{ extra marks each}
68 + 3 = 71
The mean for 7 students is 71%.
Write your solution to each problem using words, numbers, pictures, and/or models. Explain your solution using only words.

1. A large piece of construction paper is 0.01 mm thick. It is cut in half and one piece is placed on the other to make a pile. These two pieces are cut in half and all four pieces are placed in a pile. The process continues. After the pieces have been cut and piled 10 times, what is the height of the pile in centimetres?

2. A magician opened a show at the mall. On Day 1, 50 people attended the show. On Day 2, there were 78 people. On Day 3, there were 106 people. This pattern continues. When will there be at least 200 people in the audience?

3. This pentagon has each vertex connected with every other. How many triangles are in the pentagon?

4. A ball is dropped from a height of 2.30 m. After each bounce, it reaches 40% of its height before the bounce.
   a) What height does it reach after each of the first four bounces?
   b) After how many bounces does the ball reach a height of about 1 cm?

5. The Great Lakes are some of the largest freshwater lakes in the world. The areas are listed in the chart at the left. About what percent of the total area is each lake?

<table>
<thead>
<tr>
<th>Lake</th>
<th>Area (km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior</td>
<td>82 100</td>
</tr>
<tr>
<td>Michigan</td>
<td>57 800</td>
</tr>
<tr>
<td>Huron</td>
<td>59 600</td>
</tr>
<tr>
<td>Erie</td>
<td>25 700</td>
</tr>
<tr>
<td>Ontario</td>
<td>18 960</td>
</tr>
</tbody>
</table>

6. How many breaths have you taken in your lifetime?

7. A bag contains ten marbles. One marble is red, two are blue, three are white, and four are black. A marble is picked at random. What is the probability of picking:
   a) red?
   b) not white?
   c) neither black nor red?
   d) either black or white?
6.5
Surface Area of a Cylinder

Focus: Develop and use the formula for the surface area of a cylinder.

Explore

Work with a partner.
You will need a cardboard tube, scissors, and sticky tape.
Cut out two circles to fit the ends of the tube.
Tape a circle to each end of the tube.
You now have a cylinder.
Find a way to calculate the surface area of the cylinder.

Reflect & Share

Share your method for finding the surface area with that of another pair of classmates.
Work together to write a formula for the surface area of any cylinder.

Connect

A cylinder has height 10 cm and radius 4 cm.
To find the surface area of the cylinder, we think of its net.
The bases of the cylinder are 2 congruent circles.
The curved surface of the cylinder is a rectangle.

The height of the rectangle is equal to the height of the cylinder.
The base of the rectangle is equal to the circumference of the base of the cylinder, which is: $2\pi(4) = 8\pi$ cm
So, the surface area of the cylinder is:
$$SA = \text{area of 2 congruent circles + area of rectangle}$$
$$= 2(\pi(4)^2) + 10 \times 8\pi$$
$$= 32\pi + 80\pi$$

Use a calculator.
Key in: $32 \times \pi \neq 80 \times \pi$ ENTER to display $351.8583772$
$SA \approx 351.858$
The surface area of the cylinder is approximately $352 \text{ cm}^2$.

To find a formula for the surface area of any cylinder, we can apply this idea to a cylinder with height $h$ and radius $r$.
Sketch the cylinder and its net.

On the net:
The height of the rectangle is $h$.
The base of the rectangle is the circumference of the base of the cylinder: $2\pi r$
The surface area of the cylinder is:
$SA = 2(\pi r^2) + 2\pi rh$
$SA = 2\pi r^2 + 2\pi rh$
If a cylinder is like a cardboard tube, and has no circular bases, its surface area is the curved surface only: curved $SA = 2\pi rh$

**Example**
A manufacturer produces a can with height $7 \text{ cm}$ and diameter $5 \text{ cm}$. What is the surface area of the can, to the nearest square millimetre?

**Solution**
The radius of the can:
$r = \frac{5 \text{ cm}}{2} = 2.5 \text{ cm}$
Use the formula for the surface area of a cylinder.
$SA = 2\pi r^2 + 2\pi rh$
Substitute: $r = 2.5$ and $h = 7$
$SA = 2\pi (2.5)^2 + 2\pi (2.5)(7)$
Key in: $2 \times \pi \times 2.5 \times \pi \times 2.5 \times 7$ ENTER to display $149.225651$
$SA \approx 149.2257$
The surface area of the can is $149.23 \text{ cm}^2$ to the nearest square millimetre.
Give each area to the nearest square unit, unless stated otherwise.

1. Calculate the curved surface area of each tube.
   a) 
   b) 
   c) 

2. Calculate the surface area of each cylinder.
   a) 
   b) 
   c) 

3. A cylindrical tank has diameter 3.8 m and length 12.7 m. What is the surface area of the tank?

4. Cylindrical paper dryers are used in pulp and paper mills. One dryer has diameter 1.5 m and length 2.5 m. What is the area of the curved surface of this dryer?

5. A wooden toy kit has different painted solids. One solid is a cylinder with diameter 2 cm and height 14 cm.
   a) What is the surface area of the cylinder?
   b) One can of paint covers 40 m². How many cylinders can be painted with one coat of paint?

6. **Assessment Focus** A soup can has diameter 6.6 cm. The label on the can is 8.8 cm high. There is a 1-cm overlap on the label. What is the area of the label?

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**Number Strategies**

Evaluate.
• \( \frac{3}{2} + \frac{7}{12} \)
• \( \frac{3}{2} - \frac{7}{12} \)
• \( \frac{3}{2} \times \frac{7}{12} \)
• \( \frac{3}{2} \div \frac{7}{12} \)

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**Reflect**

How is the formula for the surface area of a cylinder related to the net of the cylinder? Include a diagram in your explanation.
What Do I Need to Know?

✓ **Measurements in a Circle**

The distance from the centre to the circle is the *radius*.  
The distance across the circle, through the centre, is the *diameter*.  
The distance around the circle is the *circumference*.

✓ **Circle Formulas**

In a circle, let the radius be \( r \), the diameter \( d \),  
the circumference \( C \), and the area \( A \).

Then  
\[ d = 2r \]  
\[ r = \frac{1}{2}d \]  
\[ C = \pi d, \text{ or } C = 2\pi r \]  
\[ d = \frac{C}{\pi} \]  
\[ A = \pi r^2 \]

\( \pi \) is an irrational number that is approximately 3.14.

✓ **Cylinder Formulas**

Let the height of a cylinder be \( h \) and its radius \( r \).

The volume of a cylinder is:  
\[ V = \pi r^2 h \]  
The curved surface area of a cylinder is:  
Curved \( SA = 2\pi rh \)  
The total surface area of a cylinder is:  
\[ SA = 2\pi r^2 + 2\pi rh \]
UNIT 6: Circles

What Should I Be Able to Do?

LESSON 6.1
1. a) Mark two points on a piece of paper. Join the points. Use this line segment as the radius of a circle. Draw the circle. What is its diameter?
   b) Mark two points. Join the points. Use this line segment as the diameter of a circle. Draw the circle. What is its radius?

2. Trace a large circular object. Explain how to find the radius and diameter of the circle you have drawn.

3. The diameter of a circle is 43 mm. What is its circumference? Give the answer to the nearest millimetre and to the nearest centimetre.

4. The circumference of a large crater is 219.91 m. What is the radius of the crater to the nearest centimetre?

5. The radius of a circle is 12 m. What is its area? Give the answer to the nearest square centimetre.

6. The diameter of a circular mirror is 28.5 cm. What is the area of the mirror? Give the answer to the nearest square millimetre.

7. Choose a radius. Draw a circle. What if you halve the radius?
   a) What happens to the circumference?
   b) What happens to the area? Explain.

8. A goat is tied to an 8-m rope in a field.
   a) What area of the field can the goat graze?
   b) What is the circumference of the area in part a?

9. A can of creamed corn has a label that indicates its capacity is 398 mL. The height of the can is 10.5 cm. The diameter of the can is 7.2 cm.
   a) Calculate the capacity of the can in millilitres.
   b) Give a reason why the answer in part a is different from the capacity on the label.

10. A piece of sculpture comprises 3 cylindrical columns. Each column has diameter 1.2 m. The heights of the columns are 3 m, 4 m, and 5 m. The surfaces of the cylinders are to be painted. Calculate the area to be painted. (The base each column sits on will not be painted.)
1. Draw a circle. Measure its radius.
   Calculate its diameter, circumference, and area.

2. Calculate the circumference and area of each circle.
   Give each answer to the nearest unit or square unit.
   a) b) c)

3. Arleen has 50 m of plastic edging.
   She uses the edging to enclose a circular garden.
   a) What is the circumference of the garden? Explain.
   b) What is the radius of the garden?
   c) What is the area of the garden?
   d) Arleen fills the garden with topsoil to a depth of 15 cm.
     What is the volume of topsoil in the garden?

4. What if you are asked to draw a circle with circumference 100 cm.
   a) You have a compass. Explain why it is not possible for you to
      draw a circle with this circumference.
   b) Draw a circle whose circumference is as close to 100 cm
      as possible.
      i) What is this circumference?
      ii) What is the radius of this circle?

5. Which has the greater volume?
   • a piece of paper rolled into a cylinder lengthwise, or
   • the same piece of paper rolled into a cylinder widthwise
   Justify your answer. Include diagrams in your answer.
Have you noticed how many designs use circles?

You will use circles to create a design. Your design must include:
- a circle with area approximately 80 cm²
- a circle with circumference approximately 50 cm

Your design may include:
- lines
- curves
- more than the two required circles

Here are some questions to consider:
- Will I use colour?
- What is the scale of my design?
- Which tools will I use?

Your design is to be in one of these forms:
- logo for a company
- wallpaper
- sign for a shop
- poster for an art show
- quilt sample
- sundial
- greetings card
- another idea (approved by your teacher)

Your finished project must include:
- the design
- any calculations you made
- a written description of the design and how you drew it
How are circles and cylinders related?

Write a paragraph to explain what you learned about circles and cylinders.

Check List

Your work should show:

✅ how you estimated and calculated area and circumference
✅ correct measurements and calculations
✅ a diagram of your design
✅ a clear explanation of your design, with correct use of mathematical language